# Towards $Z_2$-protected gauge-Higgs unification

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ABSTRACT: In theories with flux compactification in eight or higher dimensions, the extra-dimensional components of the gauge field may be regarded as the Higgs field candidates. We suggest a way to protect these components from getting large tree-level masses by imposing a $Z_2$-symmetry acting on compact manifolds and background fields on them. In our scheme the infinite series of heavy KK modes naturally decouples from the light Higgs candidates, whose number is generically larger than one. We also present toy models with three families of leptons, illustrating that the Yukawa sector in our scheme is fairly strongly constrained. In one of these models, one fermion gets a tree-level mass after electroweak symmetry breaking, while two others remain naturally massless at the tree level.

## Contents

## 1. Introduction and summary

The idea that the Higgs field(s) may be identified with extra-dimensional component(s) of gauge fields — gauge-Higgs unification — is of considerable interest for many years [1–3], with more recent emphasis put on the gauge hierarchy problem and stability of the electroweak scale [4–6]. For similar number of years it is known [7, 8] that an attrative way to obtain chiral 4-dimensional fermions is to populate compact extra dimensions with topologically non-trivial background gauge fields; this can be achieved via flux compactifications. These two mechanisms, however, appear to be in potential conflict with each other, as the background gauge fields generically induce large (normal or tachyonic [9]) mass terms for their perturbations tangent to extra dimensions. As a possible way out it has been noted [5] that in the case of product compact manifolds, there may occur cancellations between different contributions to the mass terms, so that some 4-dimensional scalars — components of multi-dimensional gauge fields — may be light and even massless. Once their masses are small and tachyonic, either at the tree level or due to radiative corrections, the extra-dimensional components of the gauge field perturbations become indeed the Higgs field candidates.

In this paper we elaborate on this class of theories. We give a simple characterization of potentially light 4-dimensional scalar fields in those models with flux compactification where the geometry of extra dimensions is that of a product of two-dimensional compact manifolds. With this characterization, the number of potentially light scalar fields, their tree level masses and wave functions are straightforwardly calculable. We point out that in some cases, zero values of the tree level masses, rather than being a result of fine-tuning, occur as a consequence of a discrete symmetry $Z_2$ between the manifolds entering the product. In those cases the number of massless scalars is necessarily greater than 1. Once this discrete symmetry is slightly broken at the classical level, the scalars obtain small tree level masses, half of which are automatically tachyonic, and half are normal. In fact, the multiplicity of the Higgs field candidates and their positive-$m^2$ partners is a fairly generic property of the class of models we consider, this feature being of potential

phenomenological importance. Another generic property, which is common to models of gauge-Higgs unification in more than 5 dimensions [5, 6, 10, 11] is that the quartic self-couplings of the light scalars exist already at the tree level, unlike in the simplest versions of the Hosotani/Scherk-Schwarz mechanism where self-couplings are induced radiatively and hence are often too low [12 – 15]. We will further comment on phenomenology of our models towards the end of this paper.

We then proceed by giving concrete examples showing that it is relatively straightforward to obtain a pattern of the Higgs field and chiral fermion representations resembling that of the Standard Model. Our first example, however, illustrates one of the obstacles for utilizing our construction for model building with simple gauge groups. Namely, in this example, no Yukawa interactions between the Higgs field candidates and massless 4d fermions are generated at the tree level, and this appears to be a rather general property of the $Z_2$-invariant backgrounds in models with simple underlying gauge groups. It can be evaded if the background explicitly breaks the $Z_2$-symmetry.

The second example involves extra $U(1)$ factor in the gauge group of the multi-dimensional theory and leads to a toy model of leptons. Three left-handed and one right-handed generations are obtained from a single fermion of the underlying theory, a property analogous to the multiplication of fermion generations in models with fermion localization on topological defects with non-minimal topological numbers [16]. To obtain two more right-handed "leptons" one adds extra fermions into multi-dimensional theory. In this example, only one fermion obtains a mass upon electroweak symmetry breaking, which again illustrates how constrained is the structure of the Yukawa sector.

Our examples have only illustrative nature and by no means pretend to be close to realistic extensions of the Standard Model, although they contain the correct spectrum of the standard electroweak theory of leptons. It remains to be understood whether our construction can be used for successful model-building.

## 2. Higgs from gauge fields

In this paper we consider 8-dimensional space-time, although the scheme can be generalized to 10 and higher dimensions. Let us consider the Yang-Mills theory with the action

$$S = -\frac{1}{4g^2} \int d^8 X \ \sqrt{-G} \ \text{Tr}(F_{MN} F^{MN}) \,,$$

where $G$ is the determinant of the 8-dimensional metric. The class of models of interest to us has the 8-dimensional space-time of the product form $R^4 \times M_2 \times M_2'$, where $M_2$ and $M_2'$ are 2-dimensional compact manifolds. The local coordinates on $M_2$ and $M_2'$ are denoted by $y^m$, $m = 1, 2$ and $y^{m'}$, $m' = 1', 2'$. We will often use complex combinations $z = y^1 + iy^2, \bar{z} = y^1 - iy^2$ and similarly for $z', \bar{z}'$. By an appropriate choice of coordinates the metrics on $M_2$ and $M_2'$ can, locally, be brought to the Gaussian form,

$$ds^2 = \psi^2(z, \bar{z})dz d\bar{z}, \qquad ds'^2 = \psi'^2(z', \bar{z}')dz' d\bar{z}', \qquad (2.1)$$

We will treat $M_2$ and $M_2'$ symmetrically: whatever we say about $M_2$ will also be valid for $M_2'$. Henceforth we concentrate on $M_2$.

The background configuration

$$\bar{F}_{mn} = \frac{2\pi n}{\Omega_2} \psi^2 \varepsilon_{mn} X \tag{2.2}$$

solves the Yang-Mills equations on $M_2$. Here $\varepsilon_{12} = +1$, $n$ is a constant, $\Omega_2$ is the volume of $M_2$ and $X$ is a generator of the gauge group. With appropriate normalization of $X$, $n$ takes integer values, so that the flux is quantized. We take the background gauge field on $M_2'$ to be in the same direction $X$ in the Lie algebra; this field is characterized by an integer $n'$. All other components of the background gauge field are set equal to zero.

The bilinear part of the action for perturbations becomes

$$S = -\frac{1}{2g^2} \int d^8X \ \sqrt{-G} \ \mathrm{Tr}\{D_M V_N D^M V^N - D_M V_N D^N V^M - i\bar{F}_{MN}[V_M, V_N]\}$$

where $V_M$ are the gauge field fluctuations, so that $A_M = \bar{A}_M + V_M$, and the covariant derivative is given by

$$D_M V^N = \nabla_M V^N - i[\bar{A}_M, V^N]$$

with $\nabla_M$ being the standard Riemannian covariant derivative. We are interested in the components $V_m$ and $V_{m'}$ which are tangent to $M_2$ and $M_2'$, respectively. These are vector fields on $M_2$ and $M_2'$, respectively, while from the standpoint of $R^4$ they make KK towers of scalar fields. If some KK modes of $V_m$ and/or $V_{m'}$ are light, they become the Higgs field candidates. Let us see under which conditions this indeed happens.

For the sake of argument, let us assume that $V_m$ and $V_{m'}$ decouple from each other; this assumption will be justified *a posteriori*. Then it suffices to consider the fields $V_m$ only. It is convenient to use an orthonormal frame in the tangent space of $M_2$. We denote the indices in this frame by $a, b, \ldots = \underline{1}, \underline{2}$; underlining here signifies that the indices refer to the orthonormal frame. For the metric in the form (2.1), this simply means that $V_{\underline{1}} = \psi^{-1}V_1$, $F_{\underline{12}} = \psi^{-2}F_{12}$, etc. Note that in this notation, the backgorund fields $\bar{F}_{ab}$ and $\bar{F}_{a'b'}$ are constants on $M_2$ and $M_2'$, see eq. (2.2).

The explicit form of the bilinear action for $V_a$ is

$$S = -\frac{1}{2g^2} \int d^8X \sqrt{-G}\mathrm{Tr}\{\partial_\mu V_a \partial^\mu V^a + D_b V_a D^b V^a + D_{b'} V_a D^{b'} V^a - D_b V_a D^a V^b - i\bar{F}_{ab}[V_a, V_b]\} \tag{2.3}$$

In the complex basis in the tangent space one has $V_- = V_{\underline{1}} - iV_{\underline{2}} = \psi^{-1}V_z$, $V_+ = (V_-)^\dagger$. Likewise $D_- = D_{\underline{1}} - iD_{\underline{2}}$. For quantities in the complex basis of $M_2'$ we will use the notation $V_-'$, $D_-'$, etc.; as an example, $D_-' = D_{\underline{1}'} - iD_{\underline{2}'}$.

By inspection of the expression (2.3) one observes that potentially light modes satisfy

$$D_+ V_- = D_+' V_- = 0 \ . \tag{2.4}$$

For such fields the bilinear action simplifies to

$$S = -\frac{1}{2g^2} \int d^8X \sqrt{-G}\mathrm{Tr}\{\partial_\mu V_+ \partial^\mu V_- + iV_+[\bar{F}_{+-}' - \bar{F}_{+-}, V_-]\} \tag{2.5}$$

The key point is that the masses of the complex fields $V_\pm$ are given by the difference $(\bar{F}'_{+-} - \bar{F}_{+-})$. This is precisely the cancellation found in ref. [5]: once the background is chosen in such a way that this difference is small, the fields obeying (2.4) are light, even though each of $\bar{F}_{+-}$ and $\bar{F}'_{+-}$ is large. Mass terms for all other KK modes of $V_m$ receive contributions from the Laplacian acting on $V_\pm$ and therefore for small size $M_2$ and $M'_2$ these other modes will have large masses. In this way a finite number of light scalars on $R^4$ — the Higgs field candidates — will be separated from an infinite number of heavy KK modes.

Before discussing solutions to eq. (2.4), let us make a few remarks.

1. The fields obeying eq. (2.4) satisfy $D_a V^a = 0$. Due to this property, the light modes of $V_m$ indeed decouple from the perturbations $V_{m'}$. This justifies the assumption we made above.

2. As we already pointed out, for the background field configuration given in eq. (2.2), $F_{+-}$ and $F'_{+-}$ are constants on $M_2$ and $M'_2$. Therefore, if eq. (2.4) has several solutions of one and the same $X$-charge, masses of all these light modes are equal to each other. Furthermore, if equations

$$D'_+ V'_- = D_+ V'_- = 0 \,. \tag{2.6}$$

also have non-trivial solutions of the same $X$-charge, the light modes of $V$ and $V'$ have masses squared equal in absolute value but opposite in sign: if one of them is tachyonic, another is necessarily normal, and *vice versa*.

3. If $\bar{F}'_{+-} = \bar{F}_{+-}$, the Higgs field candidates are massless at the tree level. Equality between $\bar{F}'_{+-}$ and $\bar{F}_{+-}$ may be either due to fine tuning, or a consequence of a $Z_2$-symmetry interchanging the two manifolds $M_2$ and $M'_2$ and the background fields on them. In the latter case one has

$$n = n' \,, \quad \Omega_2 = \Omega'_2 \,.$$

We think this $Z_2$-protection is a particularly interesting property of the class of models discussed in this paper. Clearly, in the case of $Z_2$-symmetry, both $V_-$ and $V'_-$ have massless modes at the classical level. If this symmetry is slightly broken at the classical level, so that $\bar{F}'_{+-} \neq \bar{F}_{+-}$, the tree level masses of both $V_-$ and $V'_-$ are small; the number of light scalars generated in this way is larger than 1. In fact, we will see in what follows that this $Z_2$-protection mechanism naturally gives rise to fairly large number of light scalar modes (either massless, or normal and tachyonic in equal number). Alternatively, one can think of generating the masses for the Higgs candidates radiatively. We will further discuss this issue at the end of this paper.

4. Like in other gauge-Higgs unification models [5, 6, 10, 11], and unlike in the case of the Hosotani/Scherk-Schwarz mechanism, quartic interactions of the Higgs candidates are present at the tree level. The reason is that solutions of eq. (2.4) have both $V_1$ and $V_2$ non-zero, and their commutator does not vanish. Thus, the quartic

Higgs self-coupling $\lambda$ is generically of order $\lambda \sim g_4^2$, where $g_4$ is the 4-dimensional gauge coupling. This is not only relevant to phenomenology, but also ensures the self-consistency of the entire approach. Indeed, there are no topological arguments guaranteeing the stability of the background (2.2), so whether or not the 8-dimensional field configutrations we discuss are stable is a dynamical issue. For low masses of the candidate Higgs bosons, $m_H^2 \ll 1/R^2$, perturbative stability can be analysed within 4-dimensional low energy theory (KK modes have positive masses squared). Since the Higgs self-coupling does not vanish, the Higgs vacuum — and hence the entire 8-dimensional solution — is perturbatively stable. It is clear, though, that we are dealing with a metastable vacuum. Its decay rate, however, is expected to be suppressed as $\exp(-\text{const}/g_4^2)$, thus making our models potentially viable.

Let us now proceed to solving eq. (2.4), still in rather general terms. The field $V_-$ is a linear combination of the generators of the gauge algebra with complex (space-time dependent) coefficients. So, one can view this field as belonging to complexified adjoint representation of the gauge algebra. In this representation one can choose the basis $T^i$ in such a way that $[X, T^i] = q^i T^i$, where $q^i$ are real, and decompose $V_-$ as $V_- = V_-^i T^i$. Obviously, the meaning of $q^i$ is that it is equal to the $X$-charge of the corresponding Higgs candidate. We will omit the superscript $i$ in what follows.

For given $X$-charge $q$, the first and the second of eq. (2.4) read

$$\partial_{\bar{z}} V_- + (\partial_{\bar{z}} \ln \psi) V_- - iq \bar{A}_{\bar{z}}(z, \bar{z}) V_- = 0$$
$$\partial_{\bar{z}'} V_- - iq \bar{A}_{\bar{z}'}(z', \bar{z}') V_- = 0$$

These first order equations are solved by

$$V_- = \frac{1}{\psi(z, \bar{z})} \exp\left\{ iq \int d\bar{z} \bar{A}_{\bar{z}}(z, \bar{z}) \right\} \exp\left\{ iq \int d\bar{z}' \bar{A}_{\bar{z}'}(z', \bar{z}') \right\} f(z) g(z') \tag{2.7}$$

where $f$ and $g$ are holomorphic functions of their arguments. These functions should be chosen in such a way that $V_-$ is normalizable, namely,

$$\int d^2 z \, \psi^2 V_+ V_- \quad , \qquad \int d^2 z' \, \psi'^2 V_+ V_-$$

are both finite. These conditions restrict the number of permissible solutions.

To proceed further, let us assume that $M_2$ and $M_2'$ are Einstein manifolds, i.e., $R_{mn} = \text{const} \cdot g_{mn}$. Then the solutions to the Yang-Mills equations on $M_2$ and $M_2'$, with the field strength given by (2.2), are

$$A_m = \frac{n}{2} \omega_m X \, , \quad A_{m'} = \frac{n'}{2} \omega_{m'} X$$

where $\omega_m = \varepsilon_{ml} \partial_l \ln \psi$ and $\omega_m' = \varepsilon_{m'l'} \partial_{l'} \ln \psi'$ are the components of spin connections in $M_2$ and $M_2'$, respectively. The solutions (2.7) then take simple form,

$$V_- = \psi^{\frac{qn}{2}-1}(z, \bar{z}) \, \psi'^{\frac{qn'}{2}} f(z) g(z') \, .$$

The norms on $M_2$ and $M_2'$ reduce to

$$\int d^2 z \psi^2 V_+ V_- = \int d^2 z \psi^{qn} |f|^2 \tag{2.8}$$

$$\int d^2 z' \psi'^2 V_+ V_- = \int d^2 z' \psi'^{2+qn'} |g|^2 \tag{2.9}$$

Clearly, the number of solutions of finite norms is finite. As we will see momentarily, this number can be easily counted by making use of the above formulas. We note in passing that there are no normalizable solutions of zero $X$-charge $q$.

The same analysis, with obvious interchange $z \leftrightarrow z'$, etc., applies to the light modes of $V_m'$.

# 3. Examples

To illustrate the general treatment, let us give simple examples of models leading to the light scalars with quantum numbers of the Higgs field of the Standard Model. We also introduce fermions in such a way as to mimic leptons.

## 3.1 SU(4)

We begin with the gauge group SU(4) and $X = \mathrm{diag}(1, 1, 0, -2)$. The background breaks SU(4) down to $\mathrm{SU(2)}_L \times \mathrm{U(1)}_X \times \mathrm{U(1)}$ in such a way that the complexified adjoint of SU(4) is decomposed as

$$\underline{15} = \underline{3}_0 + 1_0 + \underline{1}_0 + \underline{2}_3 + \underline{2}_{-3} + \underline{2}_1 + \underline{2}_{-1} + \underline{1}_2 + \underline{1}_{-2}$$

the subscript here refers to the $X$-charge. Clearly $\underline{2}_1$ is the right candidate for the Higgs doublet, once $X$ is identified with the weak hypercharge,[1] $Y = X$.

Let us take $M_2 = S^2$ and $M_2' = S'^2$ with the radii $a$ and $a'$ respectively. The metric functions are

$$\psi = \frac{1}{1 + \frac{|z|^2}{4a^2}} \quad , \qquad \psi' = \frac{1}{1 + \frac{|z'|^2}{4a'^2}}$$

It is now straightforward to count the number of light scalars in this setup. Let us take $n > 0$ and $n' > 0$. Then, according to eqs. (2.8) and (2.9), all light scalars must have positive hypercharges. Let us specify to doublets of hypercharge $q = 1$. For convergence of the integral (2.8), $n$ should be larger than 1. The choice $f = z^m$ and $g = z'^{m'}$ yields convergent norms provided that $m = 0, 1, \dots (n-2)$, and $m' = 0, 1, \dots n'$. Thus there are $(n-1)(n'+1)$ normailzable solutions in the sector of the gauge fields tangent to $M_2$. Since the background is invariant under the rotations of the two spheres, the entire spectrum and the interactions are classified according to irreducible representations[2] of $\mathrm{SO(3)} \times \mathrm{SO(3)}'$ acting on $S^2 \times S'^2$. The light Higgs belongs to $(j = -1 + n/2, j' = n'/2)$ representation of

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[1]In our normalization the left-handed lepton doublet has weak hypercharge -1 and the right-handed electron has weak hypercharge -2.

[2]The relationship between the holomorphic basis used in this paper and spherical harmonic basis of ref. [5] can be found in ref. [17].

this group. Likewise, for $n' > 1$ there is $(n'-1)(n+1)$ light modes among the gauge fields tangent to $M_2'$ belonging to $(j = n/2, j' = -1 + n'/2)$ representation of $SO(3) \times SO(3)'$.

Altogether, there are $2(nn'-1)$ Higgs candidates. When the $Z_2$-symmetry is slightly broken at the classical level, half of these putative Higgs fields will be tachyonic and the other half will have positive mass squared, where the absolute value of $mass^2$ is given by $|n/a^2 - n'/a'^2|$.

In the case of $Z_2$-symmetric setup, when $n = n'$, the number of the light scalars is either zero (for $n, n' = 0, 1$) or at least 6 (for $n = n' = 2$). This illustrates the fact that the $Z_2$-protection mechanism leads to rather large number of light scalar fields descending from the multi-dimensional gauge field.

Let us now consider fermion zero modes in this example. We start from an 8-dimensional spinor in the anti-fundamental representation of SU(4), which after symmetry breaking by the background field is decomposed as $\bar{4} = \underline{2}_{-1} + \underline{1}_2 + \underline{1}_0$. A singlet of hypercharge 2 and a doublet have quantum numbers of left-handed positron and left-handed leptons of the Standard Model. Let us see that there are corresponding zero modes.

The 8-dimensional Dirac equation reads

$$\Gamma^A E_A^M \left( \partial_M + \frac{1}{2} \omega_{M[CD]} \Sigma^{[CD]} - i \bar{A}_M \right) \chi = 0$$

where $\Gamma^A$ are $16 \times 16$ constant Dirac matrices, $\Sigma^{[CD]} = \frac{1}{4}[\Gamma^A, \Gamma^B]$ are the generators of $SO(1,7)$ and $\omega_{M[CD]}$ are the components of the spin connection. Let us take the 8-dimensional spinor $\chi$ to be chiral,

$$i\Gamma_0\Gamma_1 \ldots \Gamma_7 \chi = +\chi \tag{3.1}$$

The zero mode equations on $M_2$ and $M_2'$ reduce to

$$(\partial_4 + \Gamma_{\underline{45}}\partial_5)\chi + \frac{1}{2}[(\partial_4 + \Gamma_{\underline{45}}\partial_5)\ln\psi](1 + iqn\Gamma_{\underline{45}})\chi = 0 \tag{3.2}$$

$$(\partial_6 + \Gamma_{\underline{67}}\partial_7)\chi + \frac{1}{2}[(\partial_6 + i\Gamma_{\underline{67}}\partial_7)\ln\psi'](1 + iqn'\Gamma_{\underline{67}})\chi = 0 \tag{3.3}$$

where $q$ is now the fermion hypercharge. Let $\epsilon = \pm 1$ and $\epsilon' = \pm 1$ be eigenvalues of $i\Gamma_{\underline{45}}$ and $i\Gamma_{\underline{67}}$, that is, chiralities on $M_2$ and $M_2'$, respectively. Then the solutions to eqs. (3.2) and (3.3) are

$$\chi = \left(1 + \frac{|z|^2}{4a^2}\right)^{\frac{1+\epsilon qn}{2}} \left(1 + \frac{|z'|^2}{4a'^2}\right)^{\frac{1+\epsilon'qn'}{2}} z_\epsilon^m z_{\epsilon'}'^{m'} \chi_{\epsilon\epsilon'}^{m,m'} \tag{3.4}$$

where $z_\epsilon = \bar{z}$ for $\epsilon = +1$ and $z_\epsilon = z$ for $\epsilon = -1$, while $\chi_{\epsilon\epsilon'}$ is a spinor independent of $z, \bar{z}, z'$ and $\bar{z}'$ and satisfying the 4-dimensional chiral Dirac equation. It follows from (3.1) that its 4-dimensional chirality is $(-\epsilon\epsilon')$. We see from (3.4) that zero modes exist for $q \neq 0$ only, and that their chiralities $\epsilon$ and $\epsilon'$ must be both negative for $q > 0$ and both positive for $q < 0$. In either case, the 4-dimensional chirality is negative. Fermions of positive 8-dimensional chirality have zero modes which are left-handed from 4-dimensional viewpoint.

The zero modes have to be normalizable, that is the following integrals have to be finite,

$$\int dz d\bar{z} \psi^2 \bar{\chi}\chi , \quad \int dz' d\bar{z}' \psi'^2 \bar{\chi}\chi .$$

This restricts the number of zero modes. In particular, there are $nn'$ zero modes of hypercharge $\pm 1$ and $4nn'$ zero modes of hypercharge $\pm 2$. We see that models considered in this paper allow for several fermionic generations originating from single multi-dimensional fermion.

The major problem with this model is that all 4-dimensional fermions have the same 4-dimensional chirality. Since the original gauge interactions do not involve charge-conjugate fermions, this means that Yukawa interactions between zero fermion modes and the light scalars — extra-dimensional components of the gauge field — are absent at least at the tree level. More generally, fermions of the same sign of hypercharge have the same 4-dimensional chiralities, which is not the case in the Standard Model. To construct a model with non-zero Yukawa couplings one has to cure this problem.

### 3.2 $\mathrm{U}(3) \times \mathrm{U}(1)$

To obtain fermions of both positive and negative 4-dimensional chiralities and non-vanishing Yukawa couplings in a $Z_2$-symmetric background, we modify the previous example by adding a $\mathrm{U}(1)$ factor to the gauge group. To avoid minor but unnecessary complications, it is convenient to consider $\mathrm{U}(3)$ instead of $\mathrm{SU}(4)$ of the previous example. Thus, in our second example the gauge group is $\mathrm{U}(3) \times \mathrm{U}(1)_{\tilde{X}}$, and $X = \mathrm{diag}(1,1,0) \in \mathrm{U}(3)$.

The treatment of light scalar doublets is the same as in the previous example, so we concentrate on fermions. We choose them to have positive 8-dimensional chirality as in (3.1) and begin with $(\underline{3}, -1)$ representation of $\mathrm{U}(3) \times \mathrm{U}(1)_{\tilde{X}}$. Upon symmetry breaking by the background, $\mathrm{U}(3) \times \mathrm{U}(1)_{\tilde{X}} \to \mathrm{U}(2) \times \mathrm{U}(1)_X \times \mathrm{U}(1)_{\tilde{X}}$, it decomposes as

$$(\underline{3}, -1) = (\underline{2}, 1_X, -1_{\tilde{X}}) + (\underline{1}, 0_X, -1_{\tilde{X}}) . \tag{3.5}$$

If one wishes to identify these fermions with left-handed lepton doublets and right-handed lepton singlets of the Standard Model, one makes the assignment of weak hypercharge $Y = X + 2\tilde{X}$. With this assignment, the Higgs doublets still have weak hypercharge $Y = 1$. Note, however, that unlike in the Standard Model, the low energy gauge group is $\mathrm{U}(2) \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{\tilde{Y}}$, where $\tilde{Y}$ is the second linear combination of $X$ and $\tilde{X}$.

Now, the trick is to populate the internal manifolds $M_2$ and $M_2'$ with both $\mathrm{U}(1)_X$ and $\mathrm{U}(1)_{\tilde{X}}$ gauge fields. This can still be done in a $Z_2$-symmetric way, with $M_2 \leftrightarrow M_2'$, $F \leftrightarrow F'$ and $\tilde{F} \leftrightarrow -\tilde{F}'$ under the $Z_2$-transformation. Due to this symmetry, the topological numbers $n, n'$ of $\mathrm{U}(1)_X$ and $\tilde{n}, \tilde{n}'$ of $\mathrm{U}(1)_{\tilde{X}}$ are related as $n' = n$, $\tilde{n}' = -\tilde{n}$. The fermion doublet effectively feels Abelian fields on $M_2$ and $M_2'$ with

$$(qn)_D = n - \tilde{n} , \quad (qn')_D = n + \tilde{n} ,$$

while for the singlet one has

$$(qn)_S = -\tilde{n} , \quad (qn')_S = \tilde{n} .$$

Clearly, $(qn)_S$ and $(qn')_S$ have opposite signs, while the signs of $(qn)_D$ and $(qn')_D$ can be made the same by an appropriate choice of $n$ and $\tilde{n}$. By making use of eq. (3.4) one finds that in that case the singlet zero modes are right-handed while the doublet ones are left handed.

Let us further specify to the simplest case

$$n = 2 , \quad \tilde{n} = 1 \tag{3.6}$$

Then there are 3 left handed zero modes $D^{m,m'}$, with $m = 0$, $m' = 0, 1, 2$ and $\epsilon_D = \epsilon'_D = -1$, where $m$ and $m'$ are the integers entering (3.4). The singlet has one right-handed zero mode $S^{0,0}$ with $\epsilon_S = +1$, $\epsilon'_S = -1$, and the Higgs candidates are $V_-^{0,m'}$, $m' = 0, 1, 2$ and $V'^{m,0}_-$, $m = 0, 1, 2$. The $SO(3) \times SO(3)'$ quantum numbers of these modes are respectively $D$: $(j = 0, j' = 1)$, $S$: $(j = 0, j' = 0)$, $V$: $(j = 0, j' = 1)$ and $V'$: $(j = 1, j' = 0)$. Upon integration over $S^2 \times S'^2$ the term

$$\bar{D} V_- (\Gamma_4 + i\Gamma_5) S$$

produces a non-zero Yukawa coupling, while the Yukawa coupling with $V'_\pm$ is forbidden by $SO(3) \times SO(3)'$ symmetry. As the Yukawa terms arise from the interactions of fermions with $U(3)$ gauge fields in the 8-dimensional theory, the Yukawa couplings are of order of the 4-dimensional gauge coupling. In fact, after integrating over $S^2 \times S'^2$ and rescaling to canonically normalized fields one finds that the Yukawa coupling is equal to $2g$ where $g$ is the 4-dimensional $U(2)$-coupling. Note that when the Higgs fields get vacuum expectation values, only one fermion obtains a mass, simply because there is only one right-handed fermion coming from (3.5).

One way to obtain a toy model of leptons is to add two fermionic $U(3)$-singlets with $\tilde{X} = -1$, again of positive 8-dimensional chirality. These singlets will form two $(\underline{1}, 0_X, -1_{\tilde{X}})$ representations and will have one right-handed zero mode each. Since they do not interact with $U(3)$ gauge fields, their Yukawa couplings will be zero at the tree level. In this way one obtains three families with lepton quantum numbers, only one of them having a tree-level mass after electroweak symmetry breaking.

Clearly, the model discussed here is far from being close to realistic. It does not contain quarks, its low energy gauge group is $U(2) \times U(1) \times U(1)$, it has global $SO(3) \times SO(3)$ symmetry[3] inherited from the internal manifold $S^2 \times S'^2$, etc. Nevertheless, this model illustrates that our construction has some features which we think are interesting.

Except for special features mentioned earlier, phenomenology of the class of models we discuss appears rather similar to other models of gauge-Higgs unification in more than 5 dimensions. Naive dimensional analysis, extended to higher dimensions [18], suggests that the UV cutoff scale $\Lambda$ of the 8-dimensional theory is determined by $\Lambda^4 \simeq l_8 g^{-2}$, where $g$ is the 8-dimensional gauge coupling and $l_8 = 3! \, 2^8 \, \pi^4$ is the 8-dimensional loop factor. In terms of the size of extra dimensions $a$ one has

$$\Lambda \simeq \frac{l_8^{1/4}}{\sqrt{4\pi g_4} a} \sim \frac{10}{a} .$$

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[3] This symmetry will be promoted to gauge symmetry when gravitational interactions are included. It would of course be absent if $M_2$ and $M'_2$ had no isometry groups.

As we already pointed out, the candidate Higgs bosons may obtain tree-level masses (normal and tachyonic), if the $Z_2$-symmetry is broken explicitly by the background. Otherwise these masses may come from higher-order operators like $l_8^{-1}\text{Tr}F^4$ and from non-local operators induced radiatively; neither of these types of contributions to the masses is forbidden by the $Z_2$-symmetry. The former contributions are estimated as $(m_H^2)_{HO} \simeq (2\pi^2 g_4^2)/(l_8 a^2)$ (for $n, n' \sim 1$), while the latter are expected to be somewhat larger, $(m_H^2)_{NL} \simeq g_4^2/(l_4 a^2)$ where $l_4 = 16\pi^2$ is the 4-dimensional loop factor. Thus, our construction belongs to the class of theories with TeV-scale extra dimensions and fairly low cutoff scale $\Lambda$, about $10\,\text{TeV}$ or somewhat higher. It remains to be explored how far one can go in model-building with this construction.

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